AI-1540 CV-19

M.A./M.Sc. (Previous)

Term End Examination, 2020-21

MATHEMATICS

(Real Analysis and Measure Theory)

Paper - II

Time : Three Hours]

[Maximum Marks : 100

[Minimum Pass Marks : 36

Note : Answer any five questions. All questions carry equal marks.

1. (a) If the $\lim\limits_{\|p\| \to 0} s(p, f, \alpha)$ exists, then prove that

    $f \in R(\alpha)$ and $\lim\limits_{\|p\| \to 0} s(p, f, \alpha) = \int_a^b f d\alpha$

    (b) Let $\alpha(x) = |x|^3$ then find the value of $\int_{-1}^2 x^5 \, d\alpha$

2. (a) Prove that the sum of an absolute convergent series does not alter with any rearrangement of terms.

    (b) Let $y : [a, b] \to R^k$ be a curve, If $c \in (a, b)$ Then prove that-

    $^\wedge\tau(a, b) = {}^\wedge\tau(a, c) + {}^\wedge\tau(c, b)$

3. (a) State and prove cavchy General principle of uniform convergence.

    (b) Show that the sequence $\{fn\}$ where $fn(x) = \dfrac{x}{1+nx^2}$

    Converges uniformly on R

4. (a) Let $\{fn\}$ be a sequence of real valued function on a metric space (x, d) which converges uniformly to the function f on x. If each fn (n=1, 2, 3 ---) is continuous on x then f is also continuous on x.

    (b) Test for uniform convergence and term by term integration of series $\Sigma = \dfrac{x}{(n+x^2)^2}$

5. (a) Find the radius of convergence of the power series.

    (i) $\sum\limits_{n>1}^{\infty} \dfrac{3^n}{\sqrt{n+1}}$

    (ii) $\sum\limits_{n>1}^{\infty} \dfrac{n!}{n!} 3^n$

    (b) state and prove Tauber's Theorem.

6. State and prove Inverse function Theorem.

7. (a) find the shortest distance from the point (3/2, 0) to the parabola $y^2 = yx$

    (b) State and prove chain rule.

8. (a) Let $\{E_n\}$ be a countable collection of sets of real numbers then prove that

    $m^\wedge(\bigcup\limits_{n=1}^{\infty} An) \le \sum\limits_{n=1}^{\infty} m^\wedge(\in n)$

    (b) Prove that a continuous function defined on a measurable set is measurable.

9. (a) State and prove Bounded convergence theorem.

    (b) Let f be a bounded function defined in [a, b] If f is Riemann integrable over [a, b], then it is lebesgue integrable and

    $R\int_a^b f(x)dx = \int_a^b f(x)dx$

10. (a) If f is absolutely continuous on [a, b] then prove that f is of bounded variation.

    (b) Let $1 \le p \le \infty$ and let f, $g \in l^p(\mu)$, then $f+g \in l^p(\mu)$ and $\|f+g\|_p \le \|f\|_p + \|g\|_p$